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Corrections to the Electron Spectrum of Hawking Radiation from Asteroid Mass Primordial Black Holes: Formalism and numerical evaluation of dissipative interactions

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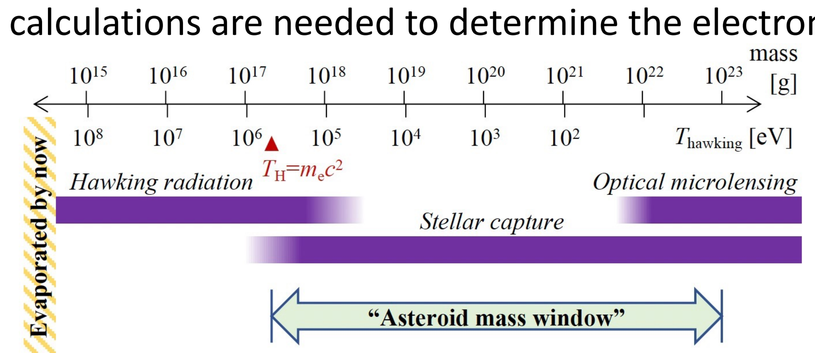
PROJECT OVERVIEW

Primordial black holes (PBHs) within the mass range of **10^{17}–10^{22} g** are regarded as a **promising dark matter candidate**. In this specific mass range, Hawking radiation serves as a crucial constraint for two main reasons: first, the Hawking temperatures are sufficiently high to facilitate the generation of gamma rays and electron-positron pairs; second, a larger number of PBHs is required to account for the observed dark matter density. In addition to constraints on PBHs arising from **gamma-ray photons** produced by Hawking radiation, positrons from PBH Hawking radiation can contribute to the **511 keV annihilation line**, imposing further constraints on PBH abundance. To model signals from high-energy observations, rigorous QED calculations are needed to determine the electron-positron spectrum from Hawking radiation.



Parameter space for low-mass PBHs. Current observations constrain their mass within the asteroid-mass window. Credit: Project Narrative, C. Hirata.

Our group focuses on determining the spectra of photons, electrons, and positrons generated by PBHs within the 10^{16}–10^{17}g range. This particular mass range has a Hawking temperature TH=1/(8πM)>100 keV, allowing PBHs to emit electron-positron pairs as the primary source of detectable Hawking radiation. Our group previously derived an analytic expression for first-order Hawking radiation spectra with dissipative effects in Schwarzschild PBHs and implemented it numerically across various black hole masses [1][2]. This study extends that work, applying a similar QED approach for a comprehensive analytical and numerical analysis of the **dissipative part of the electron spectrum**.

ANALYTICAL APPROACH

The dissipative part of electron spectrum is related the the evolution of the electron density matrix in the following way:

$$\frac{dN^{(1)}_{e^-}}{dhdt} = \frac{1}{2\pi}\sum_{km}\frac{d}{dt}\left\langle \hat{b}^\dagger_{\mathrm{out},k,m,h}\hat{b}_{\mathrm{out},k,m,h'}\right\rangle$$

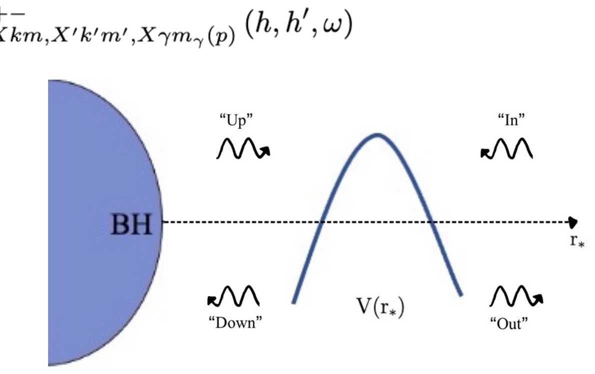
We apply **Ehrenfest's theorem** twice to derive the evolution of the electron density matrix. The first application is directly to the number operator of electron:

$$\frac{d}{dt}\left\langle \hat{b}^\dagger_{X,k,m,h}\hat{b}_{X',k,m,h'}\right\rangle = -i\left\langle\left[\hat{b}^\dagger_{X,k,m,h}\hat{b}_{X',k,m,h'},H\right]\right\rangle$$

The first-order correction to the evolution of the electron density matrix arises from the interaction Hamiltonian (shown below), which was derived in our group's previous work [1] following the canonical quantization of the electromagnetic and spinor fields.

$$H_{\rm int}(t) = e\sum_{X_\gamma \ell m_\gamma p}\int\frac{d\omega}{2\pi}\hat{a}_{X_\gamma,\ell,\omega,m_\gamma,(p)}\int\frac{dhdh'}{(2\pi)^2}\sum_{XkmX'k'm'}\Big[\hat{b}^\dagger_{Xkmh}\hat{d}^\dagger_{X'k'm'h'}I^{++}_{Xkm,X'k'm',X_\gamma\ell m_\gamma(p)}(h,h',\omega)$$
$$+\hat{d}_{Xkmh}\hat{b}_{X'k'm'h'}I^{--}_{Xkm,X'k'm',X_\gamma\ell m_\gamma(p)}(h,h',\omega)+\hat{b}^\dagger_{Xkmh}\hat{b}_{X'k'm'h'}I^{+-}_{Xkm,X'k'm',X\gamma m_\gamma(p)}(h,h',\omega)$$
$$+\hat{d}_{Xkmh}\hat{d}^\dagger_{X'k'm'h'}I^{-+}_{Xkm,X'k'm',X_\gamma\ell m_\gamma(p)}(h,h',\omega)\Big)\Big] + \text{ h.c.}$$

Where X represents a mode from the "in" (in from infinity) or "up" (up from the horizon) channel, forming a complete basis. The "down" and "out" basis are their time-reversed counterparts.



A diagram illustrating the **in/up** and **out/down** basis. Credit: [1]

We apply Ehrenfest's theorem again to compute the expectation value of the commutator between the electron number operator and the interaction Hamiltonian. In this calculation, we use the non-interacting approximation and apply Wick's theorem to decompose terms involving four-fermion and boson operators, which is justified since we are considering corrections at **O(α)**. The result consists of a combination of the phase space density g/f, three-mode overlap integrals I (describing "vertices" between photons and fermions interactions), and the Φ function:

$$\Phi(\Omega) = \frac{1}{\pi}\lim_{t\to\infty}\int_0^t e^{i\Omega(t-t')}dt' = \int_{-\infty}^t e^{i\Omega(t-t')}dt' = \delta(\Omega)+\frac{i}{\pi}\mathrm{P}(\frac{1}{\Omega})$$

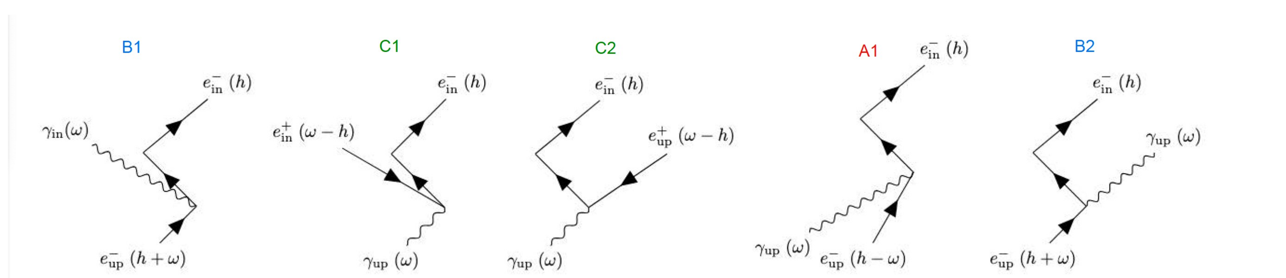
Here, δ(Ω) corresponds to the **dissipative interaction**, where the number of particles is not conserved, while P(1/Ω) represents the conservative interaction, which requires renormalization. Addressing this renormalization is part of our group's future work.

After dropping unphysical term, the dissipative part correction turns out to be

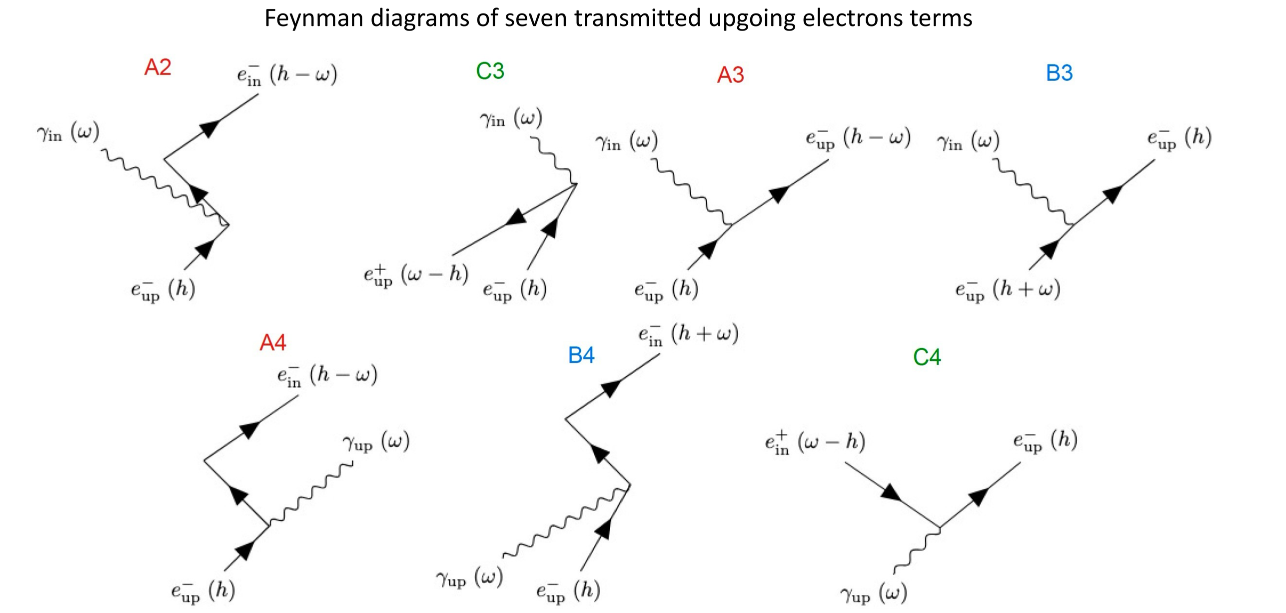
$$\frac{d}{dt}\left\langle \hat{b}^\dagger_{X_1,k,m,h}\hat{b}_{X_2,k,m,h}\right\rangle_{\rm diss} = \frac{-\pi e^2}{(2\pi)^2}\int d\omega\sum_{\ell,p}\sum_{X_\gamma,X_3\in\{\rm in,up\}}\sum_{k_3}\frac{\Delta(j,j_3,\ell)}{2\ell+1}\delta_{ss_3(-1)^{k+k_3+\ell},(-1)^p}\{$$
$$g^{e^-}_{X_1X_1k}(h)g^{e^+}_{X_3X_3k_3}(h')[\![I^{++*}_{X_1k,X_3k_3,X_\gamma\ell(p)}(h,h',\omega)]\!][\![I^{++}_{X_2k,X_3k_3,X_\gamma\ell(p)}(h,h',\omega)]\!]|_{h'=\omega-h}$$
$$+f^{e^-}_{X_3X_3k_3}(h')g^{e^-}_{X_1X_1k}(h)[\![I^{+-*}_{X_1k,X_3k_3,X_\gamma\ell(p)}(h,h',\omega)]\!][\![I^{+-}_{X_2k,X_3k_3,X_\gamma(p)}(h,h',\omega)]\!]|_{h'=h-\omega}$$
$$-f^{e^-}_{X_2X_2k}(h)g^{e^-}_{X_3X_3k_3}(h')[\![I^{+-*}_{X_3k_3,X_2k,X_\gamma\ell(p)}(h',h,\omega)]\!][\![I^{+-}_{X_3k_3,X_1k,X_\gamma\ell(p)}(h',h,\omega)]\!]|_{h'=h+\omega}$$
$$+g^{\gamma}_{X_\gamma X_\gamma\ell(p)}(\omega)[(f^{e^-}_{X_1X_1k}(h)-g^{e^+}_{X_3X_3k_3}(h'))$$
$$\times[\![I^{++*}_{X_1k,X_3k_3,X_\gamma\ell(p)}(h,h',\omega)]\!][\![I^{++}_{X_2k,X_3k_3,X_\gamma\ell(p)}(h,h',\omega)]\!]|_{h'=\omega-h}$$
$$+(f^{e^-}_{X_1X_1k}(h)-f^{e^-}_{X_3X_3k_3}(h'))$$
$$\times[\![I^{+-*}_{X_1k,X_3k_3,X_\gamma\ell(p)}(h,h',\omega)]\!][\![I^{+-}_{X_2k,X_3k_3,X_\gamma\ell(p)}(h,h',\omega)]\!]|_{h'=h-\omega}$$
$$+(f^{e^-}_{X_2X_2k}(h)-f^{e^-}_{X_3X_3k_3}(h'))$$
$$\times[\![I^{+-*}_{X_3k_3,X_2k,X_\gamma\ell(p)}(h',h,\omega)]\!][\![I^{+-}_{X_3k_3,X_1k,X_\gamma\ell(p)}(h',h,\omega)]\!]|_{h'=h+\omega}]\}$$
$$+(\text{c.c.},X_1\leftrightarrow X_2)$$

After expanding over the basis of both fermions and photons, the expression results in 22 terms. Among them, 5 terms correspond to reflected electron contributions, sharing a common factor of $|R_{1/2,k,h}|^2$; 7 terms represent transmitted electron contributions, with a common factor of $|T_{1/2,k,h}|^2$; and 10 terms arise from interference effects, characterized by the common factor $\mathrm{Re}(T_{1/2,k,h}{}^*R_{1/2,k,h})$. Their Feynman diagrams are shown below and the right column.

Feynman diagrams of five reflected ingoing electrons terms



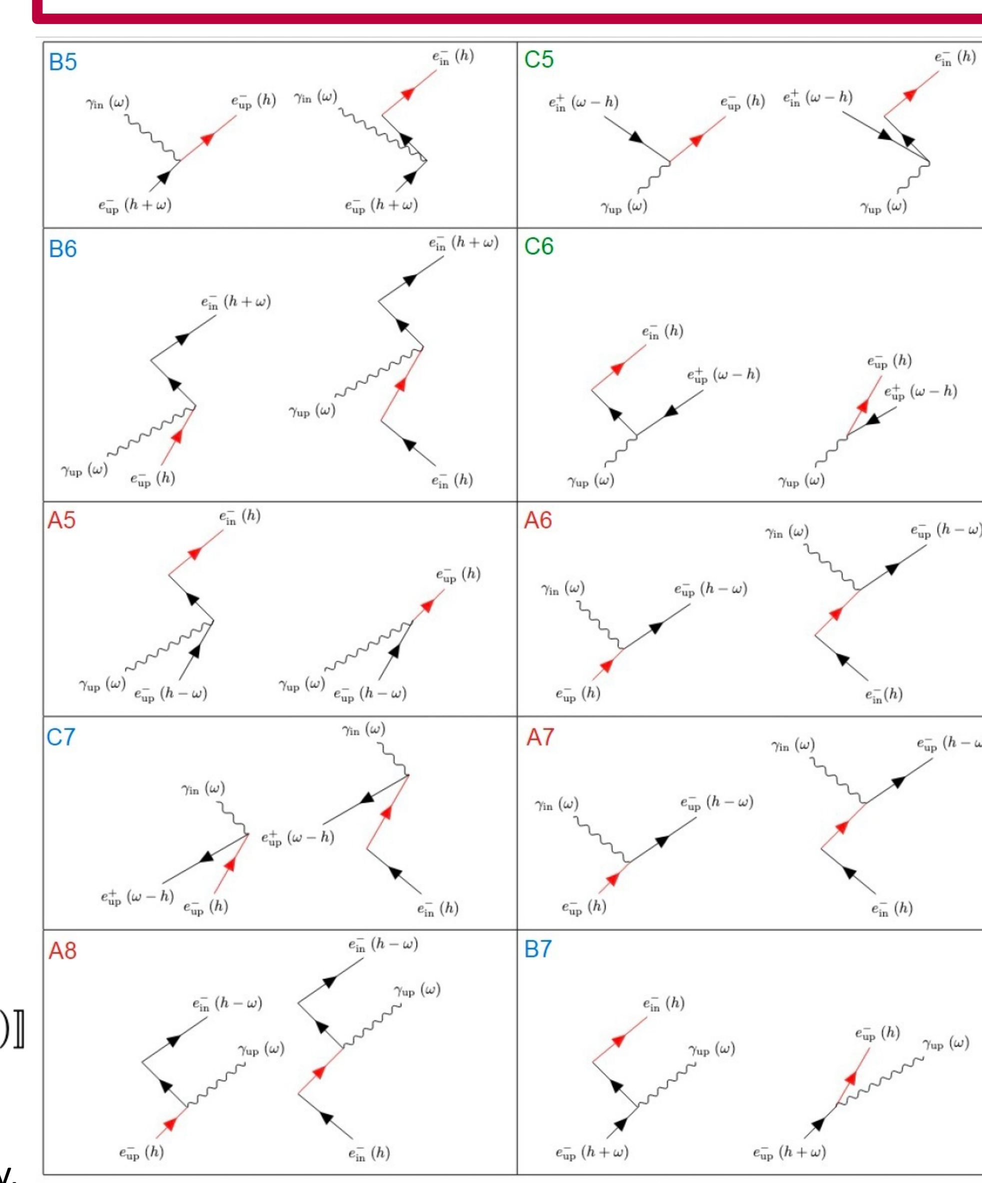
Feynman diagrams of seven transmitted upgoing electrons terms



These 22 terms can be categorized into three distinct groups based on their physical processes, as reflected by the form of the corresponding three-mode overlap integrals—Bremsstrahlung/Inverse Bremsstrahlung (A and B) and pair production and annihilation (C):

A1 – A8: $[\![I^{+-}(h,h-\omega,\omega)]\!]$ B1 – B7: $[\![I^{+-}(h+\omega,h,\omega)]\!]$ C1 – C7: $[\![I^{++}(h,\omega-h,\omega)]\!]$

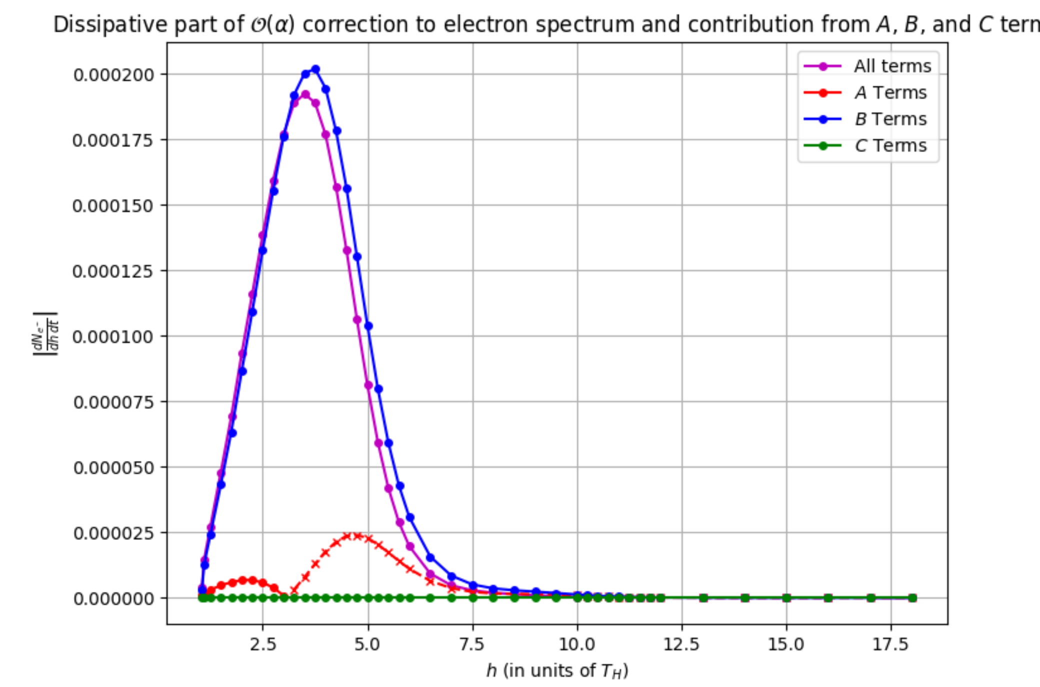
We carry out the numerical calculation for each of the 22 terms independently, computing them for each fermion quantum number and photon mode separately, while also splitting them by parity.



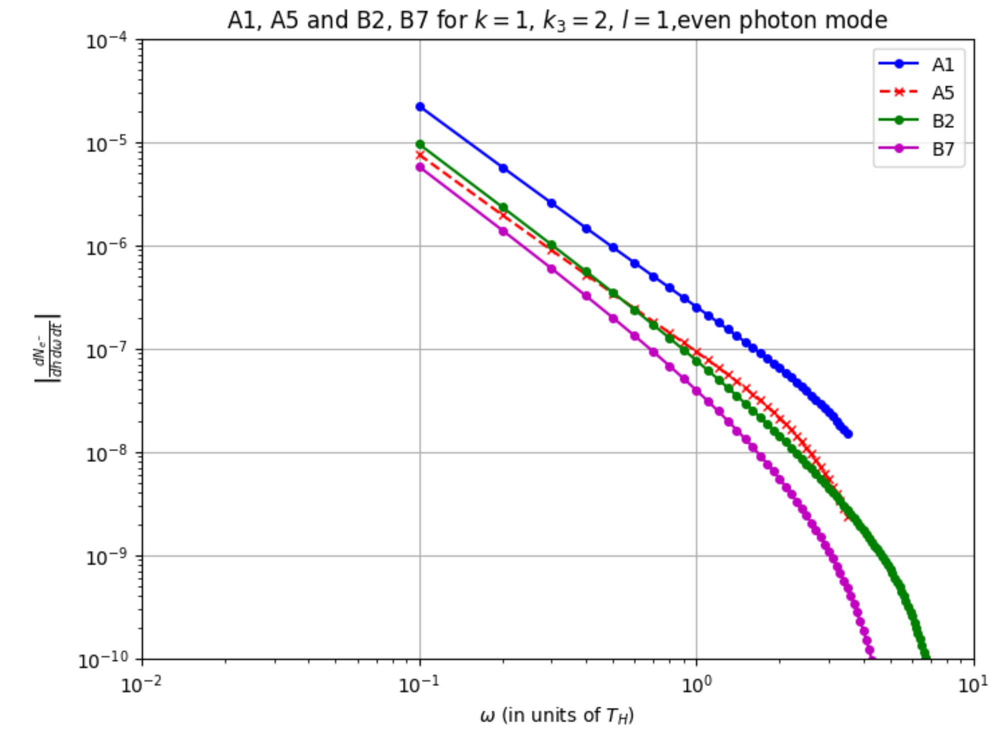
Feynman diagrams of ten interference terms

PRELIMINARY NUMERICAL RESULTS

We have conducted a preliminary run for a black hole with a mass of 10^{21} Planck masses (~10^{16}g), and the resulting spectral correction is shown in the right figure. The dashed line represents negative values. The results indicate that in the low-electron-energy region, Bremsstrahlung and Inverse Bremsstrahlung processes dominate the electron spectrum, which aligns with our expectations. We are particularly interested in understanding why terms A and B exhibit different behaviors, even though they share structural similarities and some of their terms become identical in the low photon energy limit.



Therefore, we also examine how the correction spectrum, as a function of electron energy, is distributed over photon energy ω. Specifically, we analyze the contribution of different low ω values to the correction at h~3.5T_H. It turns out that four terms—A1, A5 and B2, B7—dominate the low-ω region. These four terms exhibit a 1/ω^2 **infrared (IR) divergence** for even photon modes, and this divergence cannot be eliminated simply by summing over all fermion and photon modes. The plot on the right illustrates this behavior for a specific case with k=1, k_3 = 2, and l=1. We propose that the IR divergence in the dispersive part might be canceled by the IR divergence in the conservative part of the correction, based on the **optical theorem**.



FUTURE WORKS

Further investigation into the IR divergence is worthwhile, and the full and systematic computation of the dispersive part of the first-order correction will be conducted once the IR divergence is fully understood. Meanwhile, the study of the conservative part is also being prioritized. Additionally, our group is actively working on related projects, including the exchange interaction terms for the electron/positron spectra, led by Cara Nel, and the stochastic charge effects in primordial black holes, investigated by Gabriel Vasquez[3].

REFERENCE

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[2] E. Koivu, J. Kushan, M. Silva, G. Vasquez, A. Das, and C. M. Hirata, Phys. Rev. D 111, 045011 (2025).
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